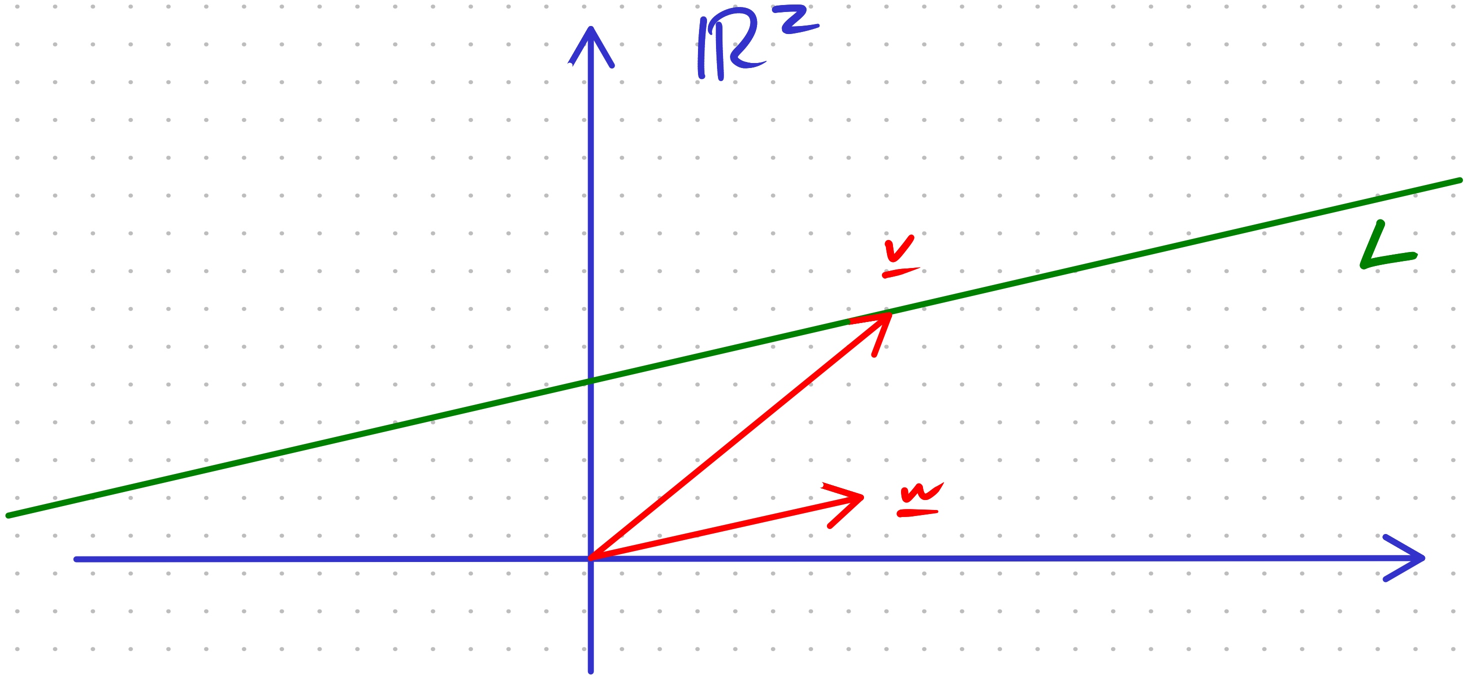
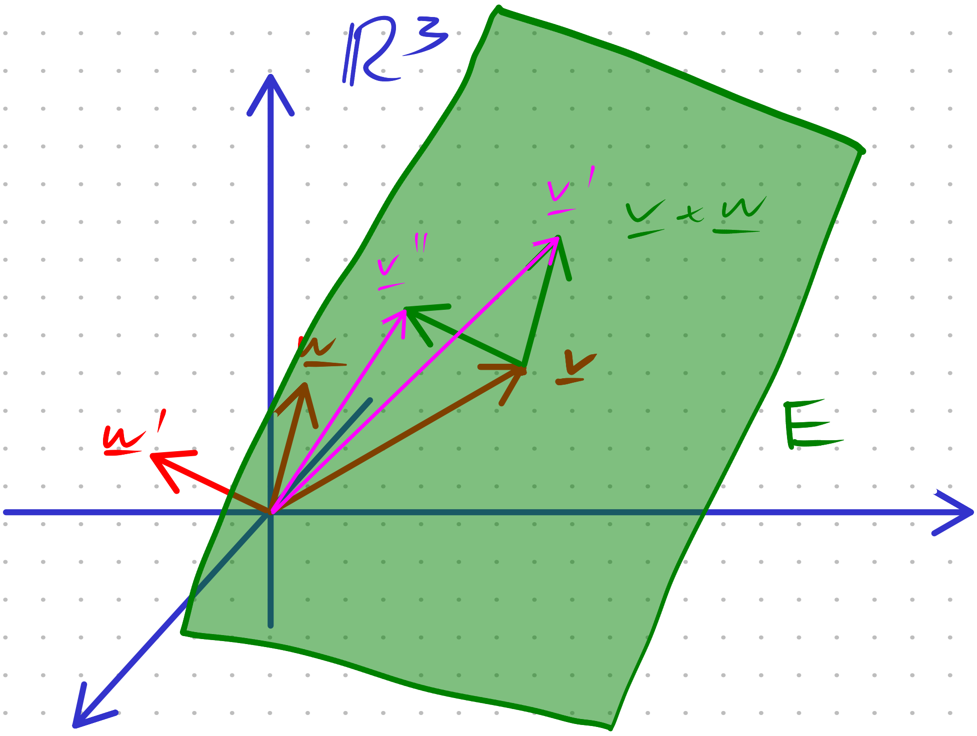



**Beweis:**

($\Leftarrow$) $a_1, a_2, a_3, b$ gegeben $\left(\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \neq \underline{0}\right)$

Falls $a_1 \neq 0$:

$$\underline{v} = \begin{pmatrix} b/a_1 \\ 0 \\ 0 \end{pmatrix} \in E$$

$$\underline{v}' = \begin{pmatrix} \frac{b-a_2}{a_1} \\ 1 \\ 0 \end{pmatrix} \in E$$

$$\underline{v}'' = \begin{pmatrix} \frac{b-a_3}{a_1} \\ 0 \\ 1 \end{pmatrix} \in E$$

$$\underline{w} := \underline{v}' - \underline{v} = \begin{pmatrix} -a_2/a_1 \\ 1 \\ 0 \end{pmatrix}$$

$$\underline{w}' := \underline{v}'' - \underline{v} = \begin{pmatrix} -a_3/a_1 \\ 0 \\ 1 \end{pmatrix}$$

Beh.: $\left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R} \;\; a_1x_1 + a_2x_2 + a_3x_3 = b \right\} = \underline{v} + \mathbb{R} \cdot \underline{w} + \mathbb{R} \cdot \underline{w}'$

($\subseteq$) ...

($\supseteq$) ...

(wie vorheriger Beweis)

Beispiel:

$$\underline{v} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \qquad \underline{w} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \qquad \underline{w}' = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$$

$$\begin{aligned} a_1 \phantom{{}+ 3a_2} &+ 2a_3 = b \\ 3a_1 + a_2 &+ 2a_3 = b \\ a_1 + 3a_2 &+ 4a_3 = b \end{aligned}$$

//

$$\begin{aligned} &(\mathrm{I}) & 2a_1 + a_2 &= 0 \\ &(\mathrm{II}) & 3a_2 + 2a_3 &= 0 \\ &(\mathrm{III}) & a_1 + 2a_3 &= b \end{aligned}$$

Wähle z.B. $a_3 = 1$

Dann $a_2 = -2/3$ (wegen $\mathrm{II}$),

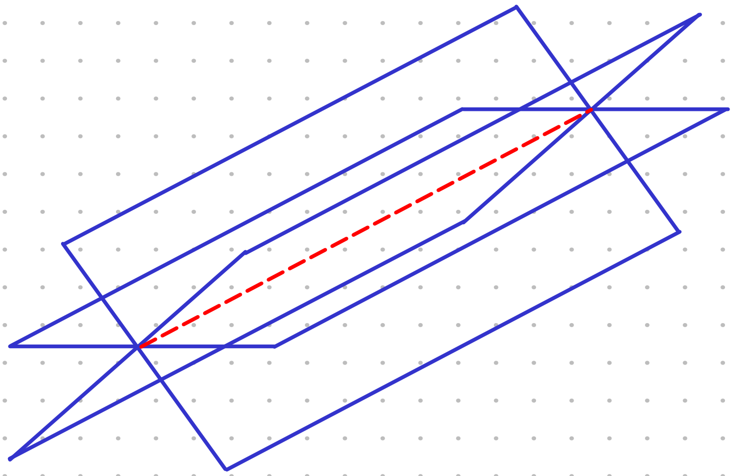
und $a_1 = -\frac{1}{2} \cdot a_3$ (wegen $\mathrm{I}$),
$= 1/3$

und $b = 1/3 + 2$ (wegen $\mathrm{III}$).
$= 7/3$

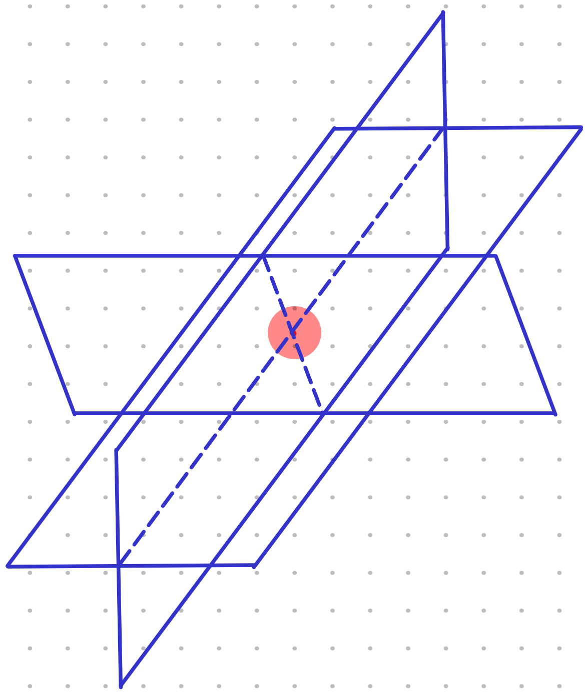
$$E = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \;\middle|\; 1/3\, x_1 - 2/3\, x_2 + x_3 = 7/3 \right\}$$

# 3 Ebenen

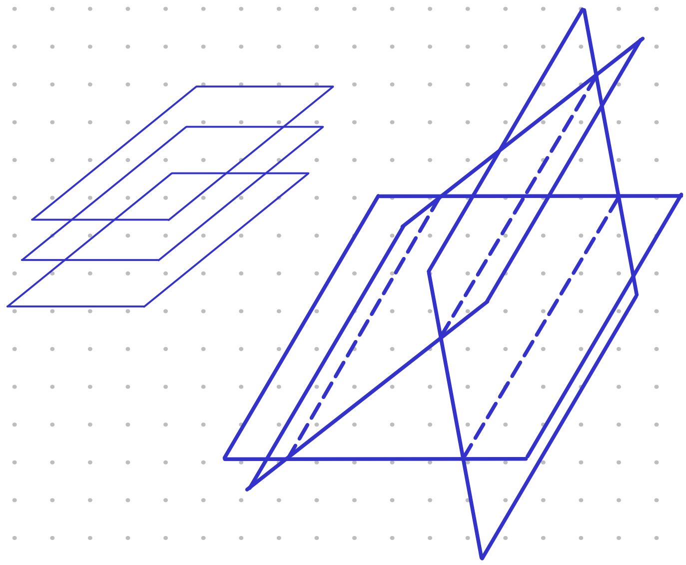
$E_1 = E_2 = E_3$



$E_1 \cap E_2 \cap E_3$ Gerade



$E_1 \cap E_2 \cap E_3$ Punkt



$E_1 \cap E_2 \cap E_3 = \emptyset$

## Beispiel:

$$E_1 = \{ \underline{x} \in \mathbb{R}^3 \mid x_1 + x_2 + x_3 = -6 \}$$

$$E_2 = \{ \underline{x} \in \mathbb{R}^3 \mid x_1 + 2x_2 + 3x_3 = -10 \}$$

$$E_3 = \{ \underline{x} \in \mathbb{R}^3 \mid x_1 - x_2 + x_3 = 1 \}$$

$$E_1 \cap E_2 \cap E_3$$

$$= \left\{ \underline{x} \in \mathbb{R}^3 \;\middle|\; \begin{array}{lrcl} & x_1 + x_2 + x_3 & = & -6 \quad (-1) \\ \text{und} & x_1 + 2x_2 + 3x_3 & = & -10 \\ \text{und} & x_1 - x_2 + x_3 & = & 1 \end{array} \right\}$$

$$= \left\{ \underline{x} \in \mathbb{R}^3 \;\middle|\; \begin{array}{lrcl} & x_1 + x_2 + x_3 & = & -6 \quad (-1) \\ \text{und} & x_2 + 2x_3 & = & -4 \\ \text{und} & x_1 - x_2 + x_3 & = & 1 \end{array} \right\}$$

$$= \left\{ \underline{x} \in \mathbb{R}^3 \;\middle|\; \begin{array}{rcl} x_1 + x_2 + x_3 & = & -6 \\ x_2 + 2x_3 & = & -4 \quad \cdot 2 \\ -2x_2 & = & 7 \end{array} \right\}$$

$$= \left\{ \underline{x} \in \mathbb{R}^3 \;\middle|\; \begin{array}{rcl} x_1 + x_2 + x_3 & = & -6 \\ x_2 + 2x_3 & = & -4 \\ 4x_3 & = & -1 \end{array} \right\}$$

Also

$$x_3 = -\tfrac{1}{4}$$

$$x_2 = -4 - 2x_3 = -4 - 2\left(-\tfrac{1}{4}\right) = -\tfrac{7}{2}$$

$$x_1 = -6 - x_2 - x_3 = -6 - \left(-\tfrac{7}{2}\right) - \left(-\tfrac{1}{4}\right) = -\tfrac{9}{4}$$

$$E_1 \cap E_2 \cap E_3 = \left\{ \begin{pmatrix} -9/4 \\ -7/2 \\ -1/4 \end{pmatrix} \right\}$$

---

$$E_1 = \{ \underline{x} \in \mathbb{R}^3 \mid x_1 + x_2 = 1 \}$$

$$E_2 = \{ \underline{x} \in \mathbb{R}^3 \mid x_1 + 2x_2 - x_3 = 2 \}$$

$$E_3 = \{ \underline{x} \in \mathbb{R}^3 \mid 3x_1 + 4x_2 - x_3 = \cancel{4}\, 3 \}$$

$E_1 \cap E_2 \cap E_3$ wird beschrieben durch:

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_1 + 2x_2 - x_3 &= 2 \\ 3x_1 + 4x_2 - x_3 &= \cancel{4}\, 3 \end{aligned}$$

$(-1)$ $(-3)$

$$
\begin{aligned}
&(\mathrm{I}) & x_1 + x_2 &= 1 \\
&(\mathrm{II}) & x_2 - x_3 &= 1 \\
&(\mathrm{III}) & x_2 - x_3 &= \cancel{1}\, 0
\end{aligned}
$$

$$
\begin{aligned}
&(\mathrm{I}) & x_1 + x_2 &= 1 \\
&(\mathrm{II}) & x_2 - x_3 &= 1 \\
&(\mathrm{III}) & 0 &= -1
\end{aligned}
$$

$$E_1 \cap E_2 \cap E_3 = \emptyset$$